Higher Dimensional Field Theories

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Quantum Field Theories

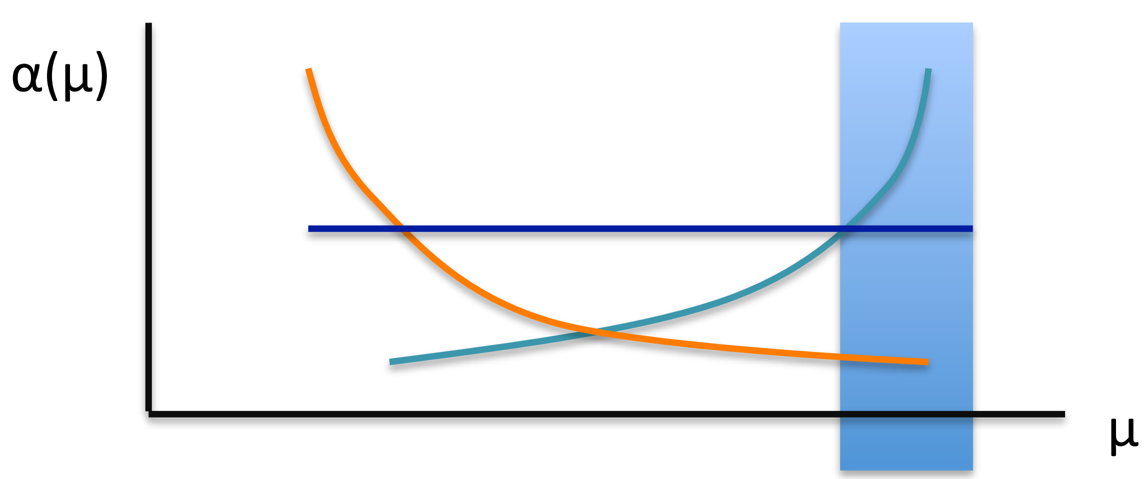
- Theories of Identical Particles
- James Clerk Maxwell: Atoms in Encyclopedia Britannica, 9th Ed. (year 1875) Atoms have been compared by Sir J. Herschel to manufactured articles, on account of their uniformity. The uniformity of manufactured articles may be traced to very different motives on the part of the manufacturer. In certain cases it is found to be less expensive as regards trouble, as well as cost, to make a great many objects exactly alike than to adapt each to its special requirements. Thus, shoes for soldiers are made in large numbers without any designed adaptation to the feet of particular men. In another class of cases the uniformity is intentional, and is designed to make the manufactured article more valuable.
- Dirac (1927): quantization of electromagnetic field by using quantum harmonic oscillators and introducing photon as field quanta

Quantum Field Theories

- Powerful Tool for studying the law of Nature
  - Space-time symmetry: Poincare symmetry, CTP
  - Gauge symmetry and internal symmetry
    - Spontaneous symmetry breaking, Higgs mechanism
  - Bose-Einstein and Fermi-Dirac statistics
  - Perturbative expansion and renormalization, RG flow, effective theory
- QED: photon+ electron
- Standard model of elementary particles
  - QCD: (1927): strong interaction, quarks and gluons SU(3) color gauge symmetry
  - Weinberg-Salam model: electro-weak theory SU(2) x U(1)
- Supersymmetry?
- Grand Unification Theory?

3 types of QFTs in 4d

- ultra-violet free or asymptotic free theories: uv complete, QCD:
- ultra-violet gets stronger with Landau pole: uv incomplete, QED, an effective theory
- ultra-violet finite with conformal symmetry: uv complete



Quantum Field Theories

- QFT as a modern calculus (Seiberg)
  - enormously useful in particle physics, condensed matter physics, cosmology and string theory/mathematics
  - do not know the right formulation
  - sign of a deep idea

- QFT as the calculus V
  - I: integration
  - II: differentiation
  - III: multivariable calculus: partial differential and integral
  - IV: infinite variable differentiation: calculus of variation
  - V: infinite variable integration: quantum field theory

Quantum Field Theories

- How to extend QFTs
- Lower dimensions (0,1,2=1+2,3=1+2)
- Higher dimensions (5=1+4,6=1+5)

5 dim & 6 dim

- Yang-Mills theory: $[1/g^2] = M$ (5d), M^2 (6d)
  - coupling expansion = $1/M$ or $1/M^2$ expansion
  - IR free, UV strong, Landau pole?
- No well defined QFTs in higher dimensions?

5 dim N=1 SUSY

- Seiberg (9608111): 5d N=1 Super Yang-Mills theories with SU(2) gauge group and n fundamental hypers have a UV fixed point with enhanced global symmetry $E_{n+1} \supset SO(2N_f) \times U(1)_I$

- instantons: the number of 1/2 BPS instantons is $U(1)_I$ charge

- Morrison, Seiberg (9609070): For n=0, one can have two options, $\theta=0,\pi$, with E_1, $\bar{E}_1$ enhanced symmetry. Non-Lagrangian theory of rank 1 E_0 without any global symmetry.

- E_0, $\bar{E}_1=U(1)$, $E_1=SU(2)$, $E_2=SU(2)\times U(1)$, $E_3=SU(3)\times SU(2)$, $E_4=SU(5)$, $E_5=SO(10)$, E_6, E_7, E_8

- $\tilde{E}_8$: SU(2) + 8 hyper: completed in 6d (1,0) SCFT

6d (1,0) SCFTs

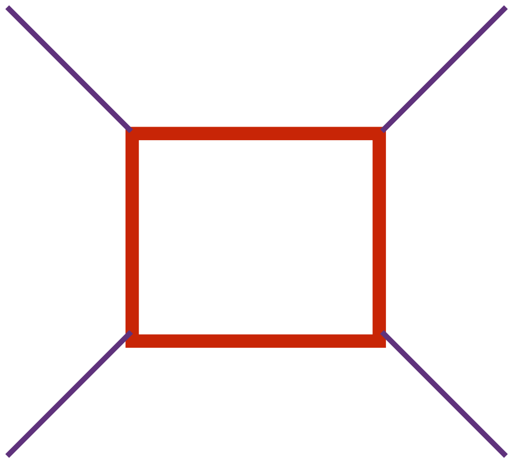
Seiberg'96,Danielsson et.al.'97

- Seiberg (9609161): Nontrivial fixed points of the renormalization group in six-dimensions
- vector multiplet (A_μ,λ)
- hyper multiplet (ϕ,ψ)
- tensor multiplet (B,Ψ,Φ)
- fermion helicity
- gauge anomalies

	helicity
vector	(0,1)
hyper	(1,0)
tensor	(1,0)
Q	(1,0)

Heckman,Morrison,Vafa(Heckman('13),Morrison, Rudelius, Vafa ('15)

Gauge Anomaly



$$\mathrm{Tr}_R F^4 = \alpha_R \mathrm{tr} F^4 + c_R (\mathrm{tr} F^2)^2$$

$$\alpha_{\mathrm{tot}} = (\alpha_{\mathrm{adj}} - \sum_{\mathrm{hyper}\ R} \alpha_R) = 0$$

$$c_{\mathrm{tot}} = (c_{\mathrm{adj}} - \sum_{\mathrm{hyper}\ R} c_R) \geq 0$$

- The gauge anomaly polynomial is made of two pieces.
  - The first one should vanish by vector and hyper contributions.
  - The second one can be removed with coupling to tensor multiplet and using the Green-Schwartz mechanism

$$H^2 + \sqrt{c_R}(B \wedge \mathrm{tr} F \wedge F + \Phi F^2)$$

6d (2,0) SCFTs

- Witten[9503124], Strominger[9512059], Klebanov and Tseytlin[9604089]
- type IIB on ADE ALE space
- theory on N M5 branes, M5+OM5
- tensor multiplet (B,Ψ,Φ_I) $\gamma^6\,\Psi=\Psi$, $H=dB={}^*H$
- M2 branes between M5 branes=selfdual strings

6d (1,0), (1,1), (2,0) LSTs

- Seiberg[9705221], Losev, Moore Shatashvili[9707250]

- (1,1) & (2,0) LSTs on NS5 branes with $g_s \to 0$, l_s: fixed
- (1,1) LST as the UV completion of 6d (1,1) SYM

- (1,0) LSTs: Bhardwaj, Del Zotto, J. Heckman, Morrison, Rudelius, Vafa [1511.05565]

Today

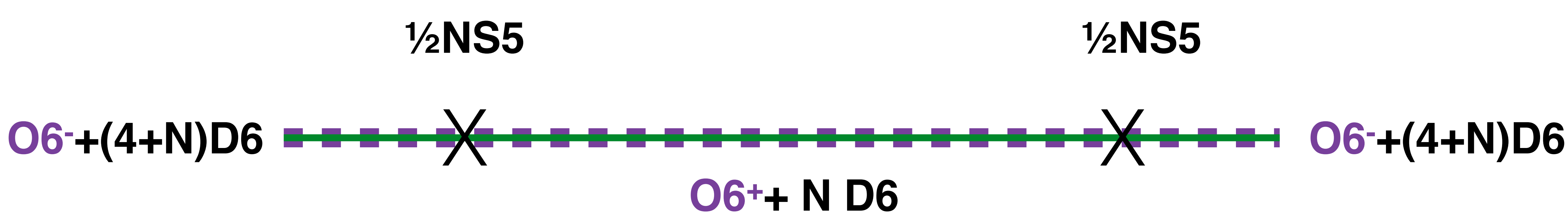
- Focus on a very small part of this theory space
- generalization of 5d Seiberg's result of SU(2) theory to larger gauge group
- relate that to 6d (1,0) SCFTs

Start with 6d (1,0) SCFTs

- Seiberg: 6d (1,0) SCFT with E_8 global symmetry
- $[E_8]$-T : M5 brane near M9 Horava-Witten Wall
- $[E_7]$-T-$[SU(2)]$: part of E_7 conformal matter
- $[E_6]$-T-$[SU(3)]$: part of E_6 conformal matter
- $[G_2]$-T-$[F_4]$: part of E_8 conformal matter
- $[SO(8)]$-T-$[SO(8)]$: D_4 conformal matter

D_{4+k} conformal matter

- Physics of a single M5 brane exploring D_{4+k} singularity $(\mathbb{C}^2/\Gamma_{D(4+k)})$
- [SO(2k+8)]-Sp(k)$_T$-[SO(2k+8)]
- Sp(k)+ tensor + (2k+8) fundamental hyper
- global symmetry = SO(4k+16)
- for k=0, SO(16) get enhanced to E_8

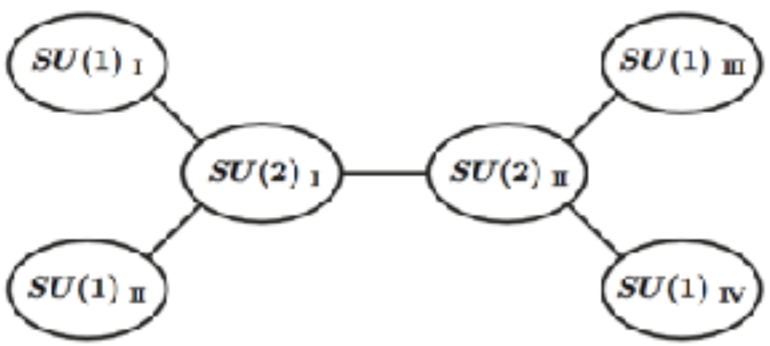


5d reduction

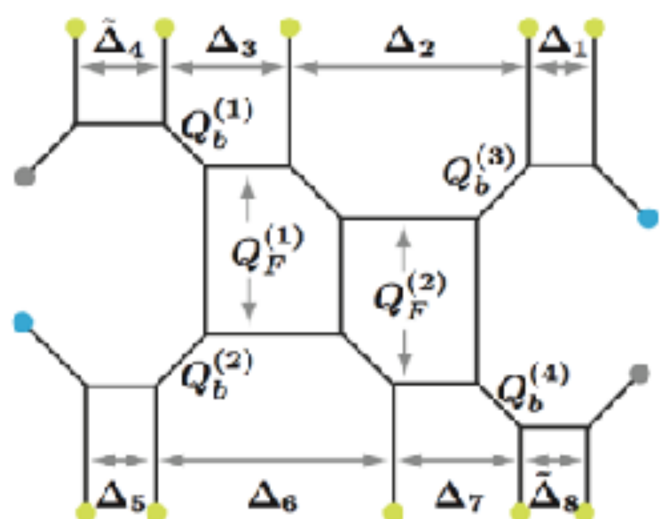
Hyashi,S.S.Kim,KL,Taki,F.Yagi 2015,Yonekura2015

- 6d: M5 brane exploring D_{k+4} singularity

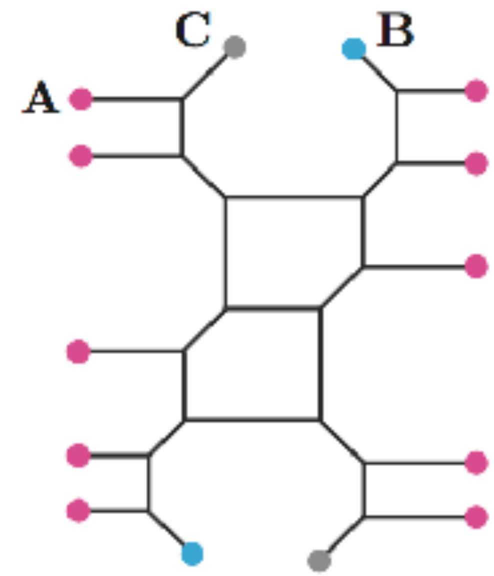
- circle compactification: a single D4 brane exploring D_{k+4} singularity = D_{k+4} type quiver theory



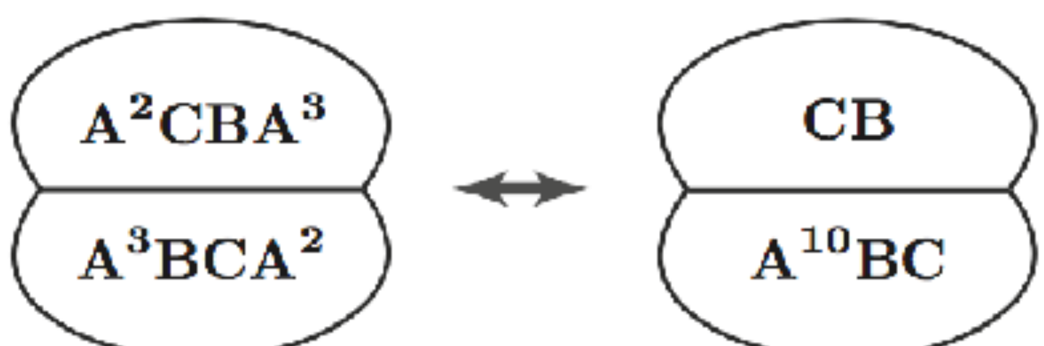
- [p,q] 5 brane web



5d reduction



- S-dual of the [p,q] brane web
- 5d SU(3) gauge theory with 10 fundamental hypermultiplets and zero CS level: global symmetry $U(10)\times U(1)_I$
- Enhanced global symmetry: $SO(20)\times U(1)_{KK}$
- 7-brane argument



$D^{10}=A^{10}BC$

5d $SU(3)_\kappa$ + N_f fund hyper

- removing hyper induce the Chern-Simons level
- rank of global symmetry= $N_f + 1$

N_f	$G_{\lvert\kappa\rvert}$ (κ is the Chern-Simons level)
10	$SO(20)_0$
9	$SO(20)_{\frac{1}{2}}$
8	$SU(10)_0$, $\quad [SO(16)\times SU(2)]_1$
7	$[SU(8)\times SU(2)]_{\frac{1}{2}}$, $\quad SO(14)_{\frac{3}{2}}$
6	$[SU(6)\times SU(2)\times SU(2)]_0$, $\quad SU(7)_1$, $\quad SO(12)_2$
5	$[SU(5)\times SU(2)]_{\frac{1}{2}}$, $\quad SU(6)_{\frac{3}{2}}$, $\quad SO(10)_{\frac{5}{2}}$
4	$SU(4)_0$, $\quad [SU(4)\times SU(2)]_1$, $\quad SU(5)_2$, $\quad SO(8)_3$
3	$SU(3)_{\frac{1}{2}}$, $\quad [SU(3)\times SU(2)]_{\frac{3}{2}}$, $\quad SU(4)_{\frac{5}{2}}$, $\quad SO(6)_{\frac{7}{2}}$
2	$SU(2)_0$, $\quad SU(2)_1$, $\quad [SU(2)\times SU(2)]_2$, $\quad SU(3)_3$, $\quad SO(4)_4$
1	$SU(2)_{\frac{5}{2}}$, $\quad SU(2)_{\frac{7}{2}}$
0	$SU(2)_3$

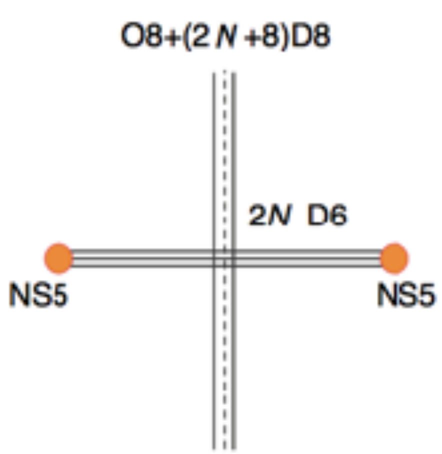
5d SU(N+1)$_\kappa$ + N$_f$ fund hyper

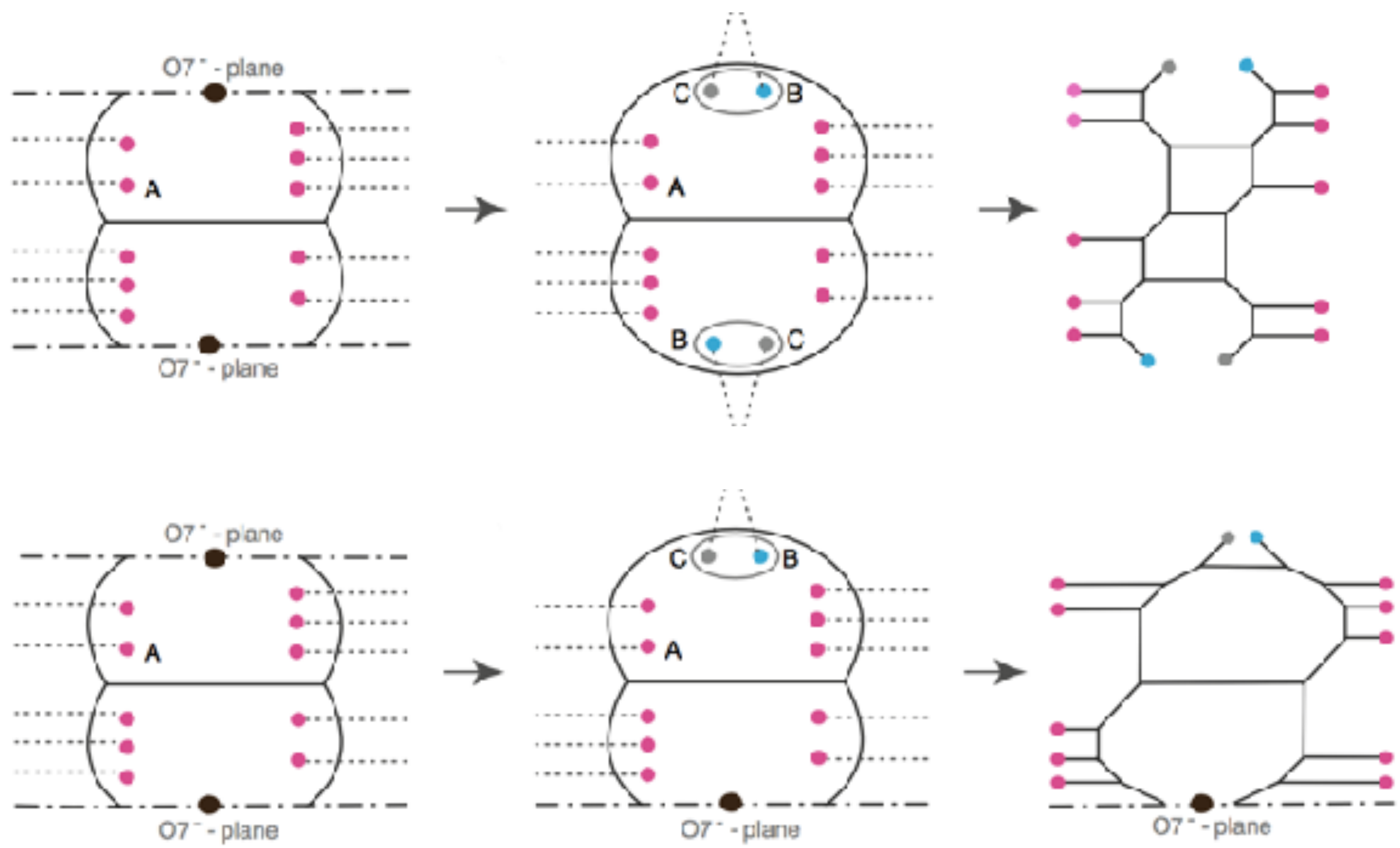
- removing hyper induce the Chern-Simons level
- rank of global symmetry= N$_f$ + 1

N_f	$G_{\lvert\kappa\rvert}$		
$2n+4$	$SO(4n+8)_0$		
$2n+3$	$SO(4n+8)_{\frac{1}{2}}$		
$2n+2$	$SU(2n+4)_0$,	$[SO(4n+4)\times SU(2)]_1$	
$2n+1$	$[SU(2n+4)\times SU(2)]_{\frac{1}{2}}$,	$SO(4n+2)_{\frac{3}{2}}$	
$2n$	$[SU(2n)\times SU(2)\times SU(2)]_0$,	$SU(2n+1)_1$,	$SO(4n)_2$

6d Sp(N)+ tensor + (2N+ 8) fund hyper

- 5d SU(N+2) + (2N+8) fundamental hyper
- 5d Sp(N+1) + (2N+8) fundamental hyper





5d duality between $SU(N)_\kappa$ & $Sp(N-1)$

Gaiotto,H.C.Kim 15

- Sp(N-1) theory: $SO(2N_f) \times U(1)$

- Enhanced global symmetry

N_f	$SU(N)_{\pm(N+1-N_f/2)}$	N_f	$SU(N)_{\pm(N+2-N_f/2)}$
$\leq 2N$	$SU(N_f+1) \times U(1)$	$\leq 2N+1$	$SO(2N_f) \times U(1)$
$2N+1$	$SU(N_f+1) \times SU(2)$	$2N+2$	$SO(2N_f) \times SU(2)$
$2N+2$	$SU(N_f+2)$	$2N+3$	$SO(2N_f+2)$

- SU(N) with $|\kappa| = N+2-N_f/2$ and N_f fund hyper = Sp(N-1) with $k=0$, N_f fund hyper

index function of dyonic instantons

- N=1 SUSY with Q invariant under j_R+R & j_L
  - 1/2 BPS dyonic instanton E= |k+q|, kq>0
- the Witten index

$$Z = \mathrm{Tr}(-1)^F e^{-\beta\{Q,Q\}} x^{2(j_R+R)} y^{2j_L} e^{-i\sum_i H_i m_i}$$

$$= Z_{\mathrm{pert}}(1+\sum_{k=1}^{\infty} q^k Z_k)$$

- instanton dynamics of Sp(N) theory = O(k) gauge group
  - $O(k)=O(k)_+ + O(k)_-$: determinant +1 or -1.

gauge holonomy of $O(k)_\pm$

- O(k)+ : $$e^{i\phi_+}=\begin{cases}\mathrm{diag}(e^{i\sigma_2\phi_1},\cdots,e^{i\sigma_2\phi_n}) & \text{for even } k\,,\\ \mathrm{diag}(e^{i\sigma_2\phi_1},\cdots,e^{i\sigma_2\phi_n},1) & \text{for odd } k,\end{cases}$$
- O(k)- : $$e^{i\phi_-}=\begin{cases}\mathrm{diag}(e^{i\sigma_2\phi_1},\cdots,e^{i\sigma_2\phi_{n-1}},\sigma_3) & \text{for even } k\,,\\ \mathrm{diag}(e^{i\sigma_2\phi_1},\cdots,e^{i\sigma_2\phi_n},-1) & \text{for odd } k,\end{cases}$$
  - $k=2n+\chi$, $\chi=0,1$
- instanton dynamics of Sp(1) theory = O(k) gauge group (ADHM data)
  - N=4 susy quantum gauge dynamics with O(k) adjoint
  - O(k)xSp(N) bifundamental,
  - U(N_f)x O(k) vector fermi multiplets
  - $O(k)=O(k)_+ + O(k)_-$: determinant +1 or -1.

gauge holonomy of O(k)$_\pm$

Kim,Kim,Lee1206.6781

- Part of k instanton partition function

$$x = e^{-\gamma_1}, y = e^{-\gamma_2}$$

$$I_+^k = (2i)^{k(N_f-2N-2)-n} i^{n+2\chi} \oint [d\phi] \left[\frac{\prod_{l=1}^{N_f} \sin\frac{m_l}{2}}{\sinh\frac{\gamma_1\pm\gamma_2}{2} \prod_{i=1}^{N} \sin\frac{i\gamma_1\pm\alpha_i}{2}} \prod_{I=1}^{n} \frac{\sin(\frac{\phi_I\pm 2i\gamma_1}{2})}{\sin\frac{\phi_I\pm i\gamma_1\pm i\gamma_2}{2}} \right]^{\chi}$$
$$\times \prod_{I=1}^{n} \left[\frac{\sinh\gamma_1}{\sinh\frac{\gamma_1\pm\gamma_2}{2} \sin\frac{2\phi_I\pm i\gamma_1\pm i\gamma_2}{2}} \frac{\prod_{l=1}^{N_f} \sin\frac{m_l\pm\phi_I}{2}}{\prod_{i=1}^{N} \sin\frac{\phi_I\pm\alpha_i\pm i\gamma_1}{2}} \right] \prod_{I<J}^{n} \left[\frac{\sin\frac{\phi_I\pm\phi_J\pm 2i\gamma_1}{2}}{\sin\frac{\phi_I\pm\phi_J\pm i\gamma_1\pm i\gamma_2}{2}} \right],$$

$$I_-^{k:odd} = \frac{(2i)^{k(N_f-2N-2)-n}}{i^{N_f-2N-n-2}} \oint [d\phi] \left[\frac{\prod_{l=1}^{N_f} \cos\frac{m_l}{2}}{\sinh\frac{\gamma_1\pm\gamma_2}{2} \prod_{i=1}^{N} \cos\frac{i\gamma_1\pm\alpha_i}{2}} \prod_{I=1}^{n} \frac{\cos(\frac{\phi_I\pm 2i\gamma_1}{2})}{\cos\frac{\phi_I\pm i\gamma_1\pm i\gamma_2}{2}} \right]$$
$$\times \prod_{I=1}^{n} \left[\frac{\sinh\gamma_1}{\sinh\frac{\gamma_1\pm\gamma_2}{2} \sin\frac{2\phi_I\pm i\gamma_1\pm i\gamma_2}{2}} \frac{\prod_{l=1}^{N_f} \sin\frac{m_l\pm\phi_I}{2}}{\prod_{i=1}^{N} \sin\frac{\phi_I\pm\alpha_i\pm i\gamma_1}{2}} \right] \prod_{I<J}^{n} \left[\frac{\sin\frac{\phi_I\pm\phi_J\pm 2i\gamma_1}{2}}{\sin\frac{\phi_I\pm\phi_J\pm i\gamma_1\pm i\gamma_2}{2}} \right]$$

$$I_-^{k:even} = (2i)^{(k-1)(N_f-2N)-\frac{5}{2}k} i^{n+4} \oint [d\phi] \left[\frac{\cosh\gamma_1}{\cosh\frac{\gamma_1\pm\gamma_2}{2} \sinh^2\frac{\gamma_1\pm\gamma_2}{2}} \frac{\prod_{l=1}^{N_f} \sin m_l}{\prod_{i=1}^{N} \sin(i\gamma_1\pm\alpha_i)} \right] \tag{3.62}$$
$$\times \prod_{I=1}^{n-1} \left[\frac{\sinh\gamma_1 \sin(\phi_I\pm 2i\gamma_1)}{\sinh\frac{\gamma_1\pm\gamma_2}{2} \sin\frac{2\phi_I\pm i\gamma_1\pm i\gamma_2}{2} \sin(\phi_I\pm i\gamma_1\pm i\gamma_2)} \frac{\prod_{l=1}^{N_f} \sin\frac{m_l\pm\phi_I}{2}}{\prod_{i=1}^{N} \sin\frac{\phi_I\pm\alpha_i\pm i\gamma_1}{2}} \right] \prod_{I<J}^{n-1} \left[\frac{\sin\frac{\phi_I\pm\phi_J\pm 2i\gamma_1}{2}}{\sin\frac{\phi_I\pm\phi_J\pm i\gamma_1\pm i\gamma_2}{2}} \right].$$

Nekrasov partition function =Gopakumar-Vafa Invariant

small charge expansion $e^{-\alpha}$

- **refined topological vertex: Iqbal,Kozcaz,Vafa [0701156]**
- **BPS degeneracy and Superconformal index: Iqbal,Vafa [1210.3605]**
- $\tilde{A} = q^{\frac{2}{8-N_f}} e^{-\alpha}$
- M-theory on non-compact CY3
- Z= exp(F)=PE(G)= $\exp\left[\sum_{n=1}^{\infty} \frac{1}{n} G_X(x^n, y^n, \tilde{A}^n)\right]$

$$G_X(x, y, \tilde{A}) = \sum_{C \in H_2(X,\mathbb{Z})} \sum_{j_L, j_R} N_C^{j_L, j_R} f(j_L, j_R) \tilde{A}^{n_C}$$

$$f(j_L, j_R; x, y) = \frac{(-1)^{2j_L+2j_R}(y^{-2j_L} + \cdots + y^{2j_L})(x^{-2j_R} + \cdots + x^{2j_R})}{((xy)^{1/2} - (xy)^{-1/2})((x/y)^{1/2} - (x/y)^{-1/2})}$$

more hypers and vectors

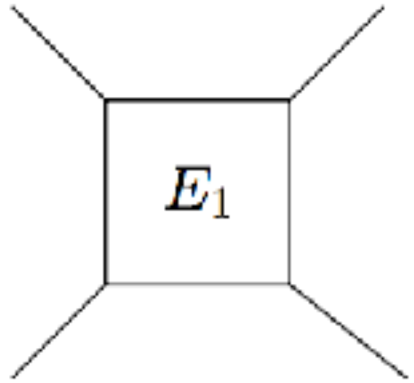
- $f(0,0)\tilde{A}$: perturbative hyper + instanton $\quad f(0,0) = \dfrac{x}{(1-xy)(1-x/y)}$
- $f(0,1/2)\tilde{A}^2$: perturbative vector + instantons $\quad f(0,1/2) = \dfrac{1+x^2}{(1-xy)(1-x/y)}$

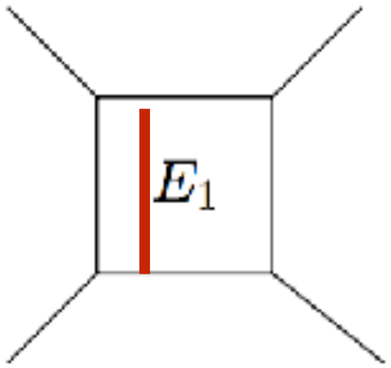
$$F_{E_6} = f(0,0)\chi_{\mu_6}\tilde{A} + f(0,\frac{1}{2})\chi_{\mu_1}\tilde{A}^2 + \left[f(0,0) + f(0,1)\chi_{\mu_2} + f(\frac{1}{2},\frac{3}{2})\right]\tilde{A}^3 + \left[f(0,\frac{1}{2})\chi_{\mu_6} + f(0,\frac{3}{2})\chi_{\mu_3} + f(\frac{1}{2},2)\chi_{\mu_6}\right]\tilde{A}^4 + \mathcal{O}(\tilde{A}^5).$$

$$\chi_{\mu_1} = 27, \quad \chi_{\mu_2} = 78, \quad \chi_{\mu_3} = \overline{351}, \quad \chi_{\mu_4} = 2925,$$
$$\chi_{\mu_5} = 351, \quad \chi_{\mu_6} = \overline{27},$$

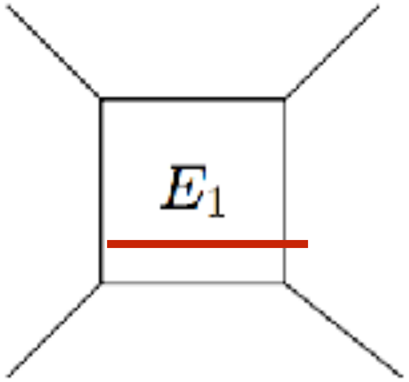
- $f(0,0)\tilde{A}$: 10+ 17 (8+1+ 8)
- $f(0,1/2)\tilde{A}^2$: 1+ 26 (8+10+8)

N_f=0





W-boson=F1



instanton=D1

0 massive hypermultiplets
2 massive vector multiplets
1 massless vector

Conclusion

- There are a lot more to be learned about 5 and 6d SCFTs and LST
- Classification of 5d SCFTs are not done yet.
- understanding the physics of SL(2,Z)
- interesting implications in lower dimensional quantum field theories
- little string theories are less explored and need more understanding.